Stationary Models of Unqueued Freeway Traffic
and Some Effects of Freeway Geometry

by

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Abstract
Occupancies and flows were jointly sampled from numerous freeway segments in nearly stationary, unqueued traffic. The data from one segment were typically averaged across all lanes there and plotted. Each plot was compared with one sampled at a neighboring freeway segment, with the two segments differing only in their number of travel lanes. Such comparisons were repeated for a total of five pairs of segments on five freeways in and near Toronto, Canada and in California. All occupancy-flow relations were piece-wise linear in form for average flows up to about 2,000 vehicles per hour per lane. Only when traffic became moderately dense did average vehicle speeds diminish with increasing occupancies. The occupancy above which these speed diminutions occurred was the same for both segments in a pair. Notably, however, the average vehicle speed corresponding to a given occupancy was always higher on the segment with the larger number of lanes. The driver psychology that can explain some of these findings is discussed. Findings are also contrasted with information currently provided in traffic handbooks.
1. Introduction

That traffic can be described by a stationary model, like that of occupancy (a dimensionless measure of density) and flow, is a common assumption in highway design and traffic analysis. Bivariate models of this kind are used, for example, in network traffic assignment (Sheffi, 1985); they are featured in recipes recommended in traffic handbooks (TRB, 2000); and they are essential parts of continuum theories of traffic flow (Lighthill and Whitham, 1955).

On one hand, these models describe relations that are apparently not independent of time. This may be due to the way drivers respond to waves that arise in traffic. Drivers do not always alter their speeds abruptly in response to waves, but instead often transition more gradually from one traffic state to the next. Waves thus exhibit characteristic widths, within which traffic conditions are not stationary, but change over time and space (Muñoz and Daganzo, 2001). Evidence suggests data from these non-stationarities do not conform to models of stationary traffic. To the contrary, these data contribute to the scatter commonly reported in bivariate data plots (Cassidy, 1998, see especially Figs. 10 and 11).

On the other hand, bivariate plots display well-defined relations (i.e., little scatter) when these consist only of average values of the data collected from sustained periods of near-stationary traffic (del Castillo and Benitez, 1995; Cassidy, 1998). The term “near-stationary” is used here to denote traffic conditions that remain constant, save for what are statistical fluctuations. An illustration of this kind of traffic is provided in the following section. A “sustained period” of nearly stationary traffic was judged in this work to be one that persisted for 4 mins or more. Earlier study found this to be a sufficient duration to average-out statistical fluctuations and produce plots that displayed little scatter (see again Cassidy, 1998).

It thus seems bivariate models can describe stationary traffic quite well. This is fortuitous, as it implies reliance on such models is reasonable, at least for describing traffic on scales of time and space that are large enough to include many vehicles.

In light of this, it makes sense to explore how certain factors, such as the number of lanes in a freeway segment, \( n \), influence the shapes of these models. And it seems logical to do this using only near-stationary traffic data. This is the objective of the work described in the present manuscript.

Some additional background and a summary of the study findings are provided in the following section. As described in section 3, scatter-plots of occupancies and flows were compared across neighboring freeway segments in ways that isolated the effects of \( n \) on the shape of the (occupancy-flow) relation. Comparisons were repeated for a number of neighboring segments and these are documented in sections 4 and 5. The findings suggest that a segment’s \( n \)
influences driver psychology in certain notable ways. This is discussed in the conclusions. Also described there are differences between the present findings and information on the subject provided in traffic handbooks.

The bivariate relations studied here were for freely flowing (i.e., unqueued) traffic only. In queued traffic, the effects of a segment’s number of lanes would not have been isolated from other influences; i.e., traffic measured in some queued segment would have been influenced by conditions downstream.

2. Background and Study Findings
The literature describes numerous studies to identify suitable forms for stationary models; the reader can consult Hall, et.al. (1992) for review of some of these. Typically, researchers have examined bivariate plots of data measured from consecutive sampling intervals of some fixed duration. As many consecutive intervals were generally used, the plots inevitably included data from non-stationary transitions in traffic states (that accompany waves). This explains at least part of the wide scatter these plots commonly displayed.

Undoubtedly, some of this scatter was also caused by having specified a priori the sampling intervals to be used. For those studies that relied upon short intervals comparable to a minute or so, the data were susceptible to statistical fluctuations caused by differences in individual driver behavior. Joint measurements over longer periods, however, surely included data points that were agglomerations of different traffic states; i.e., the time-varying values of interest were averaged-out. Data points of this latter type can create distortions on a scatter-plot.

The noise from these above sources has been diminished in the present study by sampling data in special ways and this sampling method is described in the following section. It suffices for now to note the wide scatter observed in past data plots rendered any curve-fitting exercise open to question. So a model’s form could not be estimated in any definitive way. The scatter made even more difficult the task of distinguishing relatively subtle effects of \( n \), a roadway segment’s number of travel lanes.

There is conjecture, however, concerning \( n \)’s effects on a relation’s shape. In one such theory, i) average vehicle speeds remain high so long as faster-moving vehicles can over-take slower ones; and ii) a larger \( n \) provides greater opportunity for these over-taking maneuvers (Daganzo, 1996). The theory thus states that in very light traffic, vehicle speeds are insensitive to flows. Average speeds diminish only as flows approach the roadway’s capacity. This sensitivity of speed to flow presumably depends upon the \( n \); i.e., the average flow per lane, beyond which speed diminutions begin, is higher on segments with greater \( n \).
Certain features observed in the present data are consistent with this theory. But the work has yielded additional findings described by neither the above theory nor any other. Findings are discussed below with the aid of Fig. 1.

The figure displays qualitatively the shapes of the occupancy-flow relations for data averaged across the \( n \) and relations for two different segments (with “large” and “small” \( n \)) are shown here. Verification of these shapes using the data comes later, as does description of the steps taken to isolate the effects of \( n \).

The reader will recall that the average vehicle speed corresponding to any state on the occupancy-flow plane is proportional to the slope of the chord drawn through the origin and the occupancy-flow state itself. This is illustrated in Fig. 1 with the dashed line passing through state A. The relations thus show that vehicle speeds were insensitive to flow when traffic was light.

Also as in the theory, larger \( n \) coincided with a higher threshold flow, beyond which vehicle speeds diminished. But these observed differences in threshold flows might be only a consequence of two other apparent attributes of driver behavior. First, the threshold occupancy (beyond which vehicle speeds diminish) was not influenced by \( n \). Rather, this threshold was always the same for two neighboring segments (with different \( n \)) in a pair. Second, drivers adopted higher free flow speeds on segments with greater \( n \). (These speeds are proportional to the slopes of the relations at occupancies below the threshold). Both the above features can be attributed to driver psychology discussed later.

Fig. 1 also shows that the average vehicle speed corresponding to any observed occupancy was higher on segments of greater \( n \). Notably, this was even observed for freeway geometries where a greater \( n \) did not enhance over-taking. We take this as further evidence of a certain driver psychology at play here.

Finally, the speed of a wave separating two adjacent traffic states is proportional to the slope of the line connecting these states in the occupancy-flow plane. This is illustrated in Fig. 1 with the dotted line connecting states A and B. The relations’ piece-wise linear forms thus mean that, for occupancies above the threshold, changing traffic states propagate forward as waves at speeds lower than those of the vehicles.

The procedures used to estimate relations are described in the next section. Presentations of the relations themselves come thereafter.

3. Study Scope and Methodology
The occupancies and flows used in this work were jointly extracted from nearly stationary, unqueued traffic using loop detectors on neighboring freeway segments. A segment’s \( n \) differed
from that of its neighbor, but design standards were identical for both segments in a pair. Measurements from a total of five pairs of neighboring segments were collected on freeways in and near Toronto, Canada and in California. The detectors used for measuring these data were located some distances from ramps. All measurements occurred during fair weather.

The data from one segment were typically averaged across all lanes there and the scatter-plot of these average occupancies and flows was compared with the plot from the neighboring segment. This method of pair-wise comparison was used to isolate the effects of \( n \). Observed differences in the shapes of two neighboring relations are thus attributed here solely to \( n \).

Such differences were made plainly visible by a careful method of sampling. Only average values of the data from sustained, near-stationary traffic conditions were used in the scatter-plots. This eliminated much of the noise that might have otherwise obscured subtle distinctions in neighboring relations.

To distinguish near-stationary periods from non-stationary ones, the detector data were processed in ways to obtain curves like those in Fig. 2(a). The bold line shown in the figure is a curve of \( N(x, t) \), the cumulative count of vehicles, \( N \), made by detectors at location \( x \) by time \( t \). The lighter line, \( T(x, t) \), is the curve of the detectors’ cumulative occupancy; this can be defined as the total vehicle trip time over the detectors at \( x \) by \( t \) (Lin and Daganzo, 1997).

These \( N \)- and \( T \)-curves were both measured from the same starting time. Each was re-scaled by the constants \( a \) and \( \beta \) so that they had the same numerical value at time \( t_e \), the ending time shown in the figure; \( aN(x, t_e) = \beta T(x, t_e) \).

The slopes of the piece-wise linear curves of \( N \) and \( T \) are the flows and occupancy rates, respectively. To identify visually any changes in these, a function of the form \( b \cdot t \) was subtracted from both re-scaled curves.

We explain below why the curves in Fig. 2(a) reveal that traffic conditions were nearly stationary during the period shown. First, the \( N \)-curve displays a linear trend, indicating the period was marked by nearly constant or “quasi-linear” vehicle arrivals. Moreover, both curves in the figure are nearly superimposed. Their nearly matching wiggles indicate fluctuations in flows were highly correlated with fluctuations in occupancy. It follows that the vehicles represented by these curves had nearly the same physical lengths (as seen by the detectors), \( l \), and that they all traveled at nearly the same speed, \( v \).

The rationale for this diagnosis is made clear with the aid of the time-space diagram in Fig. 2(b). The vehicle trajectories in this diagram are passing a location \( x \) and each is shown as a swath of height \( l \) and slope \( v \). The total time vehicles occupy \( x \) is the product of \( l/v \) and the number of vehicles passing \( x \). Flows and occupancies are therefore strictly proportional; i.e., a
fluctuation in the vehicle arrivals is proportional to an accompanying fluctuation in occupancy. So for conditions marked by near-constant $l$ and $v$, flow and occupancy are mutually related by a scaling factor. This is what is depicted by the nearly superimposed curves in Fig. 2(a).

The figure thus verifies the measured vehicles displayed a nearly constant flow and speed; i.e., it indicates traffic conditions at the detector location were near-stationary for the period shown. The occupancy-flow plots presented in the following two sections consist of data selected in the above manner. The sampling intervals used in this work thus varied from one observation to the next; these intervals were dictated by the periods that traffic remained nearly stationary. Periods when the re-scaled N-curve was not quasi-linear, and/or when the N- and T-curves were not nearly superimposed, were considered to be non-stationary. Data from these periods were discarded.

By using the above diagnostic to identify periods from which to extract data, any nearly stationary periods that included fast and slow vehicles and/or long and short vehicles may have been excluded unjustifiably. Since the detector data were collected over 30-sec sampling intervals, more discriminating tests could not be used.

Also of note, the diagnostic rendered simple the task of distinguishing queued from unqueued traffic states; (only the latter were used in this study). This is because the arrival of a queue’s tail to a measurement location is marked by changes in the slopes of the N- and T-curves that occur abruptly and in opposite directions (see, for example, Cassidy and Bertini, 1999).

4. Some Pair-Wise Comparisons
This section presents the scatter-plots measured on three of the five pairs of neighboring freeway segments. Plots from a fourth and fifth pair are saved for the section following this one.

**Eastbound Queen Elizabeth Way**
The presentations begin with data from abutting segments on the freeway stretch shown in Fig. 3(a), a portion of the eastbound Queen Elizabeth Way near Toronto, Canada. These data were measured during 5 weekday mornings by detectors 24 and 26; (the numbering scheme adopted here is the one used by the regional transport authority in 1995 when these data were collected). The figure shows these detectors resided on segments with 3 and 4 lanes. Both detector stations are relatively free from the influence of on- and off-ramps. Detector 24 is located some distance from its nearest ramp. The off-ramp immediately upstream of detector 26 is little used during the morning rush.
Fig. 3(b) displays the measured data. The unshaded circles are from detector 24; the shaded ones are from 26. Each set of data is of joint measurements that were averaged over all travel lanes at the respective detector station. Since the data display little scatter, the best-fit lines included in the figure describe the relations quite well. These lines indicate the relations were like those previously exemplified in Fig. 1.

Both relations in Fig. 3(b) are piece-wise linear for average flows up to about 2,000 vehicles per hour per lane (vphpl). The reader may wish to verify that the slopes of these relations change by using a straightedge (especially for the relation at 24, since its slope change is more subtle and perhaps not easily detected with the eye). The data from 24 also indicate the relation takes a non-linear form at flows higher than 2,000 vphpl. A lightly drawn dotted line is included in the figure to highlight this.

Diminutions in average vehicle speeds seem to begin at about the same average occupancy (i.e., 6.5 percent per lane) for both relations. Here again the reader might find a straightedge helpful in verifying this.

Fig. 3(b) also shows that average occupancies ranging from about 1 percent to 12 percent were measured at both detectors (while occupancies beyond 12 percent were observed only at 24). For any occupancy within this common range, the vehicle speed was higher at detector 26, the segment with larger $n$. A sizable difference in the free flow vehicle speeds is clearly evident: assuming an average vehicle length (as seen by the detector) of 6m, these speeds are estimated to be approximately 120 and 100 km/hr at detectors 26 and 24, respectively.

Southbound Interstate 5
Fig. 4(a) illustrates a stretch of southbound Interstate 5 in Los Angeles, California. Measurements at this site were made in the 4 lanes at milepost (MP) 13.35 and in the 3 lanes at MP 14.34. Data from the former MP were measured during six weekdays in 2000. Data from the latter were sampled over a singe weekday in 2001.

The occupancy-flow relations are shown in Fig. 4(b). They display features qualitatively like those shown previously. The best-fit lines indicate i) the relations are piece-wise linear for the observed range of unqueued traffic conditions; ii) the occupancy beyond which vehicle speeds diminish appears the same for both segments (i.e., 5.0 percent per lane); and iii) the average vehicle speed corresponding to any observed occupancy is higher on the 4-lane segment than on its 3-lane neighbor.

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1 In this manuscript, measurements from the segment with the larger $n$ (in each pair) will always be shown with shaded data points.
Northbound Interstate 880

Fig. 5(a) illustrates the third site used in this study; a stretch of northbound Interstate 880 near San Francisco, California. Measurements were made in the 3 regular-use lanes at detector station 12 and in the 4 regular lanes some miles downstream at detector 7. Measurements were not taken in the high-occupancy vehicle (HOV) lane, since its traffic was consistently light.

The occupancy and flow data were sampled at these detectors during portions of two days in 1993 and these are shown in Fig. 5(b). These data exhibit more scatter than do the observations from any other of our sites and the source of this scatter is unknown; (it might have been due to the detectors, the drivers’ behavior or something else). But even in the presence of this scatter, the piece-wise linear forms are clearly evident in these relations, as are the distinctions between both of them. Their features are, in short, much like those of the relations shown previously.

5. Further Evidence of \( n \)'s Effects

In this section, occupancy-flow scatter-plots are presented for neighboring segments on two additional freeways. Pair-wise comparisons reveal the effects of \( n \) to be much like those in the previous section. In these new cases, however, the segments of larger \( n \) did not seem to afford greater vehicle over-taking opportunities

Eastbound Gardiner Expressway

Fig. 6(a) displays a stretch of the eastbound Gardiner Expressway in Toronto. Detectors 20 and 30 used for measuring the data are shown as well. These data were gathered during 2 weekdays in 1999. The figure makes clear that the shoulder lane at detector 30 is a mandatory exit lane and that the off-ramp is located a fairly short distance downstream.

The data are presented in Fig. 6(b). The shaded points are flows and occupancies averaged over the median and center lanes (only) at detector 30; adjacent measurements in the shoulder lane were excluded because traffic there was very light.

It appears that at 30, the shoulder lane served exiting vehicles almost exclusively. The counts (and occupancies) made by the detector there were consistently much lower than those in the two adjacent lanes. Moreover, the (time-varying) counts in these two adjacent lanes were always comparable to those measured downstream at detector 20. It therefore appears the additional lane at 30 did little to enhance over-taking opportunities beyond those available at 20.
Yet the occupancy-flow relations estimated at these locations still display the distinctions that were evident in the previous presentations: at any observed occupancy, the vehicle speed at detector 30 is higher than on its neighboring segment with smaller $n$. Admittedly, the differences in these two relations are less pronounced than those observed at the other sites. But distinctions are evident nonetheless. They suggest that vehicle over-taking cannot, in itself, explain the effects of $n$ on the shapes of bivariate relations.

We see more evidence of other or additional such influence(s) at another freeway site. This evidence will be presented momentarily. We note first, however, that both relations in Fig. 6(b) are piece-wise linear in form. Moreover, the occupancy beyond which vehicle speeds diminish is the same for both relations (7.3 percent per lane).

Westbound Gardiner Expressway

Fig. 7(a) illustrates a portion of Toronto’s westbound Gardiner Expressway along with detectors 50 and 60 used for measuring data. The figure shows the additional lane at detector 50 is actually an acceleration lane for the on-ramp located 290m upstream. Notably, the shoulder lane counts at this detector were virtually always zero or close to zero; i.e., nearly all vehicles from the on-ramp had merged into the freeway’s adjacent through-moving lane(s) prior to arriving at 50.

Fig. 7(b) presents the data sampled at the two detector stations. These were taken during 3 weekdays in 1997. The shaded data points are measurements averaged across the three (through-moving) lanes at 50.

The two relations have piece-wise linear forms. More notably, the distinctions between them are qualitatively like those observed in all previous cases, even though the greater $n$ at 50 seems to offer no advantages in over-taking.

Alas, it is possible that some of the distinction evident in Fig. 7(b) is linked to the nearby on-ramp; (see Fig. 7(a)). Ongoing research is exploring this possibility.

6. Conclusions

The present findings indicate relations between freeway occupancies and flows are piece-wise linear in form over a wide range of unqueued conditions. In one instance, a relation was observed to be non-linear at flows very close to the segment’s capacity. These findings indicate waves can arise in unqueued traffic and travel forward at speeds lower than those of the vehicles. It follows that shocks can form in uncongested traffic. But these waves and shocks would only appear when traffic is moderately dense. In light traffic, vehicle speeds are insensitive to occupancies and flows, such that changing states are carried forward with vehicles.
Pair-wise comparisons of neighboring relations show the average vehicle speed corresponding to a given occupancy (or flow) was higher on the segment with larger $n$. It is not clear this can be attributed entirely to greater opportunities afforded fast vehicles to over-take slower ones. Notably, the higher free flow speeds observed on segments of greater $n$ are not obviously linked to over-taking. After all, a free flow speed prevails even in very light freeway traffic when, for any $n$, over-taking occurs without delay. And in denser traffic (when speeds fell below the free flow one), the vehicle speed for a given occupancy was higher on the segment of greater $n$ even where this added $n$ did not improve over-taking.

So the observed effects of $n$ on vehicle speed may have as much (or more) to do with driver psychology than with over-taking. All else being equal, drivers seem inclined to adopt a higher speed when traveling on freeway segments that are wider; i.e., on segments that have larger $n$.

Of further note, the occupancy beyond which average vehicle speeds diminished was always the same for both relations in a pair. That speeds became sensitive to occupancies above some “threshold” might be linked to vehicle over-taking. If such is the case, occupancies above the threshold coincide with vehicle spacings that are small enough to hinder over-taking maneuvers. But this observed sensitivity of speeds could be more a car-following effect whereby, in sufficiently dense traffic, drivers adjust their speeds in response to the vehicle spacings.

Finally, findings reported here are not consistent with information currently provided in traffic handbooks. The speed-flow models contained in early editions of the *Highway Capacity Manual* (TRB, 1965; 1985) are cases in point. To their credit, these models show that $n$ influences the relation in uncongested traffic. Freeway segments of larger $n$ are shown to support a higher average speed (for the same flow) and this is consistent with our present findings.

But these earlier models show vehicle speed to be a continually decreasing function of flow, even in very low flows. Such forms are contrary to the findings offered here. (Judging from some of the more recent literature, models displaying sensitivity of speeds to low flows are now regarded as out-dated or incorrect).

Models in newer editions of the *Capacity Manual* (TRB, 1994; 2000) no longer display the forms of their predecessors. Instead, vehicle speeds are insensitive to flows in light traffic. Furthermore, different models are now provided for freeway segments with different free flow vehicle speeds. But while free flow speeds are said to be influenced by design standards (i.e., roadway alignments) and speed limits, no mention is made concerning the influence of $n$ on these speeds. Yet such influence is clear in the present findings.
The *Highway Capacity Manual* is itself a widely circulated document. Moreover, the stationary models it recommends frequently find their way into other traffic handbooks (including, for example, *AASHTO, 1994*). There seems need to revisit these models in light of the findings presented here.

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